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COMPARISONS OF THEORETICAL AND EXPERIMENTAL LIFT AND PRESSURE DISTRIBUTIONS ON AIRFOILS IN CASCADE

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SUMMARY

Comparisons of theoretical with experimental airfoil lift coefficients and pressure distributions were made for two cascades of entrance vanes and three cascades of blower blades with NACA 6-series airfoils, and for one cascade of turbine blades. In general, the experimental lift coefficients were considerably less than the theoretical lift coefficients, and the differences were greater than the differences between the theoretical and experimental values for the isolated airfoils. The pressure distributions were correspondingly different; however, when the theoretical lift was made equal to the experimental lift, either by neglecting the Kutta condition or by using the Pinkerton distortion method given in NACA Rep. No. 563, fair agreement between the pressure distributions resulted. It is suggested that the aspect ratios of the blades in the cascade tests were possibly too small to avoid pronounced end effects and that the pressure distribution on the blades of an actual blower may be appreciably different from that predicted from cascade tests.

INTRODUCTION

A reasonably practical method of calculating the theoretical lift and velocity distribution on an airfoil in cascade was described in reference 1. Because this method or similar methods might find extensive application in the design of turbine or blower blading, or at least in predicting the characteristics of given turbines or blowers, a comparison between theory and experiment was considered very desirable. Accordingly, several examples were selected from among the extensive number of blower-blade and entrance-vane cascade configurations for which test results are given in references 2 and 3, and theoretical lift coefficients and velocity distributions were calculated for comparison with the data. A calculation was also made for a turbine-blading configuration for which experimental (unpublished) data were available.

In general, the theoretical lift coefficients were much larger than the experimental values, and the velocity distributions showed corresponding differences. Modifications of the basic method were therefore also tried, by which the lift coefficients were reduced to the experimental values, in order that the velocity distributions might be compared at the same lift coefficients. These various comparisons are given in the present paper, together with some discussion of the discrepancies.

SYMBOLS

c_l	section lift coefficient, based on mean velocity
Γ	circulation on airfoil
V_o	mean velocity across cascade, calculated from measured upstream velocity and measured turning angle
V_a	axial component of velocity
W_1	incoming flow velocity
W_2	outgoing flow velocity
ΔW	change in tangential component of velocity across cascade
W_2'	outgoing flow velocity corresponding to increased axial component
v	local velocity on airfoil, theoretical or calculated from local measured pressure
α_o	angle of chord line with mean flow
λ_o	angle between mean air and normal to cascade
β	angle between incoming air and normal to cascade
s	distance along airfoil surface
δ	ratio of change in tangential velocity component across cascade to mean velocity $(\Delta W/V_o)$
σ	solidity, ratio of chord to gap

CALCULATION METHODS

The basic concepts and procedures for the interference method of computing the theoretical velocity distribution on an airfoil in cascade are given in reference 1. In effect, the method fixes attention on one airfoil of the cascade (the central airfoil) and considers the velocity distribution on it to consist of two parts - that possessed by the isolated airfoil in the uniform mean flow (the vector average of the incoming and outgoing velocities) and that induced on the airfoil by the remaining airfoils of the cascade (the outer or interfering airfoils). The first part is determined by Theodorsen's method (reference 4), and the second part can be determined as described in reference 1 if the velocity distribution on the interfering airfoils is known. By successive approximations a velocity distribution is found such that, when it is used in calculating the velocity induced by the outer airfoils, the same velocity distribution results on the central airfoil. Convergence may be very rapid; for about unit solidity, it is frequently essentially complete in the first step.

Two variations of the method were used to reduce the theoretical lift coefficient to the experimental value for the purpose of comparing experimental and theoretical velocity distributions at the same lift coefficient. One procedure was merely to neglect the Kutta condition and to adjust the circulation arbitrarily so that it equals the measured circulation (the integral of $v\,ds$ around the contour, where v was obtained from the measured pressure distribution). In this way an exact potential flow about the specified airfoil is obtained, except that the rear stagnation point is on the upper surface instead of at the trailing edge. The other procedure corresponded to that recommended by Pinkerton (reference 5) in which the velocity is calculated for a distorted airfoil that is formed by curving up the rear part of the given airfoil until the theoretical lift, with the stagnation point at the trailing edge, equals the measured lift.

In the present work the distortion was applied only for calculating that part of the velocity distribution contributed by the isolated airfoil in the mean flow. In the calculation of velocities induced by the interfering airfoils, neither the interfering airfoils nor the central airfoil was considered to be distorted - specifically, the same chart readings and the same airfoil stretching factor $\frac{d\varphi}{ds}$ (see reference 1) were used as in the calculations of the normal potential flow. The inconsistency involved in this procedure was considered acceptable because of the arbitrariness, together with the inherent inconsistencies, of the distortion procedure itself;

In addition, preliminary studies of other variations of the distortion method failed to indicate any variations that would generally improve the agreement of theory with experiment. The mean flow used in these calculations was assumed, as in references 2 and 3, to be that defined by the incoming flow and the measured turning angle. For purposes of comparison with the theoretical results, which were obtained in the form of distributions of v/V_0, the experimental pressure distributions have been reduced to the same form by using the square roots of the results given in references 2 and 3.

RESULTS AND DISCUSSION

Entrance Vanes

In figure 1 are shown results for a cascade of NACA 65-810 airfoils arranged as entrance vanes, where the air enters normal to the cascade and, after turning, leaves at a higher velocity and lower pressure. The theoretical velocity distribution on the airfoil as calculated by the method of reference 1 (designated "normal potential flow" in fig. 1) is seen to be considerably different from the experimental velocity distribution; and the theoretical value of lift coefficient is 0.73 as compared with the experimental lift coefficient of 0.46, a difference of 0.27.

The discrepancy, however, must be considered as only partly a cascade effect inasmuch as the isolated airfoil, tested under otherwise similar conditions (reference 2), had a lift coefficient about 0.3 less than the theoretical value; for example, at zero angle of attack, it was only 0.48 as compared with the theoretical value of 0.77, a loss of 38 percent of the theoretical design lift coefficient. Discrepancies of this type seem to be characteristic of the NACA 6-series airfoils having an $a = 1.0$ mean line, presumably because the boundary layer blankets the curvature near the trailing edge. The usual data for less highly cambered airfoils and for higher Reynolds numbers, however, show losses closer to 25 or 30 percent.

The velocity distribution obtained by adjusting the circulation to equal the experimental value is shown in figure 1 by the short-dashed curve. Compared with the experimental results, the curve shows a somewhat larger lift toward the front (compensated by the large negative lift near the trailing edge that results from neglecting the Kutta condition) and a somewhat rounder hump on the upper surface; otherwise, the curve corresponds reasonably well with the test points. The curve obtained with the Pinkerton distortion method falls almost exactly on the test points. The amount of distortion required to make the theoretical lift coefficient agree

with the experimental value corresponded to a reduction of 0.40 in the lift coefficient of the isolated airfoil (52 percent of the design lift coefficient), which was greater than the lift reduction found in the tests of the isolated airfoil. The fact that this value is greater than the difference of 0.27 between the theoretical and experimental lift coefficients in cascade merely reflects the fact that the slope of the lift curve for the airfoil in cascade is less than for the isolated airfoil.

A similar comparison for the NACA 65-(12)10 airfoil in an entrance-vane cascade is shown in figure 2. The discrepancy between the experimental results and the normal theory was even greater than for the NACA 65-810 airfoil and the theory showed a pronounced velocity peak at the leading edge. Again, adjusting the circulation showed a reasonable agreement and the Pinkerton distortion method provided a much closer agreement on the upper surface. The amount of distortion required corresponded to a reduction of 0.66 in the lift coefficient of the isolated airfoil or 57 percent of the theoretical design lift coefficient, 1.16. This reduction is again somewhat greater than that found in the tests of the isolated airfoil (at zero angle of attack, the lift coefficient was 0.68 as compared with the theoretical value of 1.16, a reduction of 0.48 or 41 percent).

Blower Blades

In figures 3 and 4 are shown results for the NACA 65-410 and 65-810 airfoils, respectively, arranged in cascades representing blower rotors. As with the entrance vanes, considerable discrepancies exist between the curve for the normal potential flow and the test results, but a fair correspondence is obtained with either the reduced circulation or the Pinkerton distortion. For the NACA 65-410 airfoil, the required distortion corresponded to a lift-coefficient reduction of 0.28 as compared with approximately 0.20 for the isolated airfoil at about the same lift coefficient. The required distortion for the NACA 65-810 blades corresponded to a lift-coefficient reduction of 0.41, about the same as for the entrance vanes.

For the NACA 65-(12)10 airfoil (fig. 5) the discrepancy between the normal theory and the test results was very large. The required distortion corresponded to a lift-coefficient reduction of 1.01, which is twice the deficiency measured for the isolated airfoil and 50 percent greater than that for the same airfoil in the entrance-vane cascade. The velocity distribution for the distorted airfoil shows considerably more load toward the front than the experimental distribution shows, although the velocity distribution obtained by simply adjusting the circulation corresponded rather well with the test results over the forward half of the upper surface (the region of greatest interest).

Since, as has already been pointed out, airfoils having $a = 1.0$ mean lines tend to show losses of lift relative to the normal potential flow, the suggestion might be made that eliminating the camber toward the rear of the airfoil would permit improved correlation between theory and experiment. In addition to the obvious objection that correlation with theory should not be a design criterion, a further objection is that the ideal blower blade should probably have more rather than less camber toward the rear, as was indicated in reference 2. Some compromise may result, however, from considerations of practical blade construction, which may require removal of both the cusp and the curvature near the trailing edge (as in the NACA 6A-series airfoils, reference 6).

Turbine Blades

The results of a similar study for a cascade of turbine blades are shown in figure 6. Because of the large pressure drop across the cascade and the straight mean line near the trailing edge, substantial agreement between the test results and the normal theory was considered more likely to exist than for the preceding examples. The theory, however, predicted a lift coefficient of 2.5 as compared with the experimental value of 2.0, although the two velocity distributions are roughly comparable. Arbitrarily reducing the circulation or distorting the airfoil improved the agreement.

Of incidental interest is the peculiar distribution of load on the blade. Most of the load is forward of about the 65-percent-chord point, where the space between the velocity curves for the upper and lower surfaces decreases sharply and remains narrow back to the trailing edge. Apparently at such high solidities the forward part of this blade - that is, the curved part - serves to turn the air, and the straight tail of the airfoil (called "guidance") serves mainly to guarantee the exit-flow direction and to reduce the losses at high Mach numbers. This peculiarity of the loading is distinctly a cascade effect, inasmuch as the isolated airfoil shows only the usual gradual tapering off of the loading toward the trailing edge.

The peaks shown at the rear of the curve for the normal potential flow result from the finite thickness at the trailing edge. In order to avoid confusion the peaks have not been shown for the two other curves.

The absence of a sharp trailing edge leaves a certain arbitrariness in the choice of the rear stagnation point at which the Kutta condition is to be applied. In the present work the trailing-edge stagnation point was considered to be at the center of the trailing-edge arc in calculating the normal potential flow. Inasmuch as the

upper-surface boundary layer should be quite sensitive near the rear, whereas the lower-surface boundary layer has been subjected to a falling pressure over its entire length, the effective rear stagnation point is probably closer to the top of the trailing-edge arc. (Compare reference 7.) The curve for reduced circulation, for which the rear stagnation point is just below the top of the arc, may thus be considered to have much more physical significance for the turbine blade than for the previously discussed airfoils, which had sharp trailing edges.

The general accuracy of the calculations for the turbine cascade was considerably less than for the other cascades or for the examples described in reference 1. The combination of very high solidity and very high loading resulted in very large interference potentials (the potentials induced by the interfering airfoils); and since the interference velocities were determined by measuring the slopes of a faired curve of the interference potential, the absolute inaccuracy of the results was correspondingly large. In the curves of figure 6, which were obtained by three or four approximations, those irregularities that did not appear consistently in successive approximations have been faired out. An analytical method of getting the slopes would probably have been more satisfactory. Some such improvement in technique appears necessary before the method of reference 1 can be extended to the accurate design of such high-loading high-solidity cascades, even if a theory could be found for correctly predicting the experimental velocity distribution.

Possibilities of Three-Dimensional Effects

Entrance vanes and turbine blades.- A résumé of pertinent data for the six cascades studied is given in table I.

Because of the pressure drop across the entrance vanes, the boundary layers on both the vanes and the wall are expected to be thinner than the boundary layers in the tests of the isolated airfoils, and the characteristics of the entrance vanes would accordingly be expected to be closer to the theoretical characteristics. That the effective airfoil distortions exceed those of the isolated airfoils suggests that some additional cascade effect is involved. Possibly the pressure gradient across the flow passages (from the suction side of each vane to the pressure side of the adjacent vane) causes sufficient flow of wall boundary layer onto the suction sides to distort the flow seriously. Such secondary flows are frequently dominant characteristics of the flow through duct bends in which, however, both the gradients and the boundary layers are usually more pronounced than in these entrance-vane cascades.

For the turbine cascade, as previously noted, the velocity distribution indicated for the reduced-circulation case has more physical justification than for the NACA 6-series airfoils. Nevertheless, both this curve and that for the Pinkerton distortion still differ considerably from the experimental velocity distribution, so that again some three-dimensional effects probably exist. The previously mentioned secondary flows were possibly very pronounced in this cascade because of the very strong pressure gradient across the flow passages and because of the very low values of the blade aspect ratio and span-gap ratio - about 0.55 and 1.0, respectively.

Blower blades.- In table I have been included the values of blade lift coefficient that correspond by potential flow to the measured turning angles according to the formula

$$c_l = \frac{2\delta}{\sigma}$$

or

$$\frac{\Gamma}{V} = \frac{\delta}{\sigma}$$

For the blower-blade arrangements, these lift coefficients were appreciably larger than the lift coefficients corresponding to the measured pressure distributions on the blades. The question immediately arises as to how the air can be turned by an amount greater than that corresponding to the apparent circulation on the blades. The most likely reason is that there was sufficient thickening or separation of the boundary layer on the walls to reduce appreciably the effective distance between the walls downstream of the cascade. Such a reduction of the effective passage area would cause an increase in the downstream-velocity component normal to the cascade without affecting the downstream-tangential-velocity component (fig. 7) and would hence result in a deceptively large turning angle. The fact that the downstream static pressures were generally appreciably lower than would be expected on the basis of the turning angle is consistent with this explanation. In any case, the flow anomalies that presumably result from three-dimensional effects appear to be much more pronounced for the blower-blade cascade than for the entrance-vane or turbine cascades.

When the blades were tested in an actual blower, the turning angles were found to be approximately the same as those found in the cascade tests, but the pressure rise corresponded very closely to the turning angle. Presumably, then, separation or thickening of the boundary layer was less pronounced on the hub and outer shell of the blower than on the walls of the cascade tunnel. Several reasons may be suggested for this difference in behavior. Possibly centrifugal or Coriolis forces tend to throw the boundary layer outward from the hub, where the relative pressure rise (pressure rise/incoming dynamic pressure) is large, toward the tip where the relative pressure rise is less. Furthermore, the total-pressure deficiency in the upstream boundary layer tends to be greater for the stationary cascade than for the blower; because in the case of the blower the tangential component of W_1, which is introduced by the relative motion of the rotating blades, does not contribute to the relative velocity that creates this boundary layer. For example, the total-pressure deficiency at the base of the wall boundary layer just ahead of the cascade is equal to the dynamic pressure of the approaching flow $\frac{1}{2}\rho W_1^2$; whereas for the blower the total-pressure deficiency at the base of the boundary layer is merely the dynamic pressure of the axial flow $\frac{1}{2}\rho V_a^2$.

Since the pressure rise across the blower exceeded that across the cascade, the lift coefficient of the blower blades must have exceeded that of the corresponding cascade blades. Accordingly, the pressure distributions on the blades in the blower must have been appreciably different from those on the blades in the cascade. Because of the smaller distortion or circulation adjustment that would be required to duplicate the lift, agreement between theoretical and experimental pressure distributions would probably be closer for the blower than for the cascade. Measurements of the pressure distributions on the blades of an actual blower, or at least on cascade blades of increased aspect ratio, would seem very desirable.

CONCLUSIONS

Comparisons have been made between theoretical and experimental pressure distributions for two cascades of entrance vanes and three cascades of blower blades with NACA 6-series airfoils having $a = 1$ mean lines, and for one cascade of turbine blades. The main results and conclusions are:

1. The experimental lift of the NACA 6-series airfoils was considerably less than the theoretical lift.

2. For the NACA 6-series airfoils the lift deficiencies, considered in terms of effective airfoil distortions, were greater than those of the isolated airfoils, even when there was a pressure drop across the cascade.

3. For the turbine blade, the experimental lift was also considerably less than the theoretical lift; however, the theoretical lift was somewhat arbitrary because the blade did not have a sharp trailing edge.

4. A reasonable approximation to the experimental velocity distribution on the airfoil can be made if the experimental lift coefficient is known. Either arbitrarily adjusting the circulation on the airfoil or using the Pinkerton distortion method is fairly satisfactory for this purpose.

5. Three-dimensional effects, arising from the low aspect ratios of the blades, probably have considerable influence on the experimental pressure distributions, more so for the blower cascades than for the entrance-vane or turbine cascades.

6. It is possible that the pressure distributions on the blades of a rotating blower are appreciably different from those observed on the stationary cascade.

7. Increasing the aspect ratio of the blades tested in the stationary cascade may give data more nearly applicable to a blower.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
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TABLE I

SUMMARY OF DATA

Type of cascade	Airfoil	α_o (deg)	λ_o (deg)	Theoretical c_l	c_l from pressure distribution	c_l from turning angle	Δc_l required for agreement (a)	Δc_l of isolated airfoil (b)
Entrance vanes	NACA 65-810	1.8	-6.8	0.73	0.46	0.47	-0.40	-0.29
	NACA 65-(12)10	3.4	-10.8	1.11	.70	.75	-.66	-.47
Blower blades	NACA 65-410	6.5	55.5	.79	.54	.62	-.28	-.14
	NACA 65-810	4.4	54.8	.92	.56	.72	-.41	-.29
	NACA 65-(12)10	7.5	53.4	1.47	.63	.92	-1.01	-.47
Turbine blades	----------	10.2	-19.5	2.50	2.00	2.03	-.50	----------

[a]Value of Δc_l of isolated airfoil required to make theoretical lift agree with that found from experimental pressure distribution, where Δc_l is obtained by Pinkerton distortion.

[b]Difference between experimental and theoretical c_l of isolated airfoil at zero angle of attack (data from reference 2).

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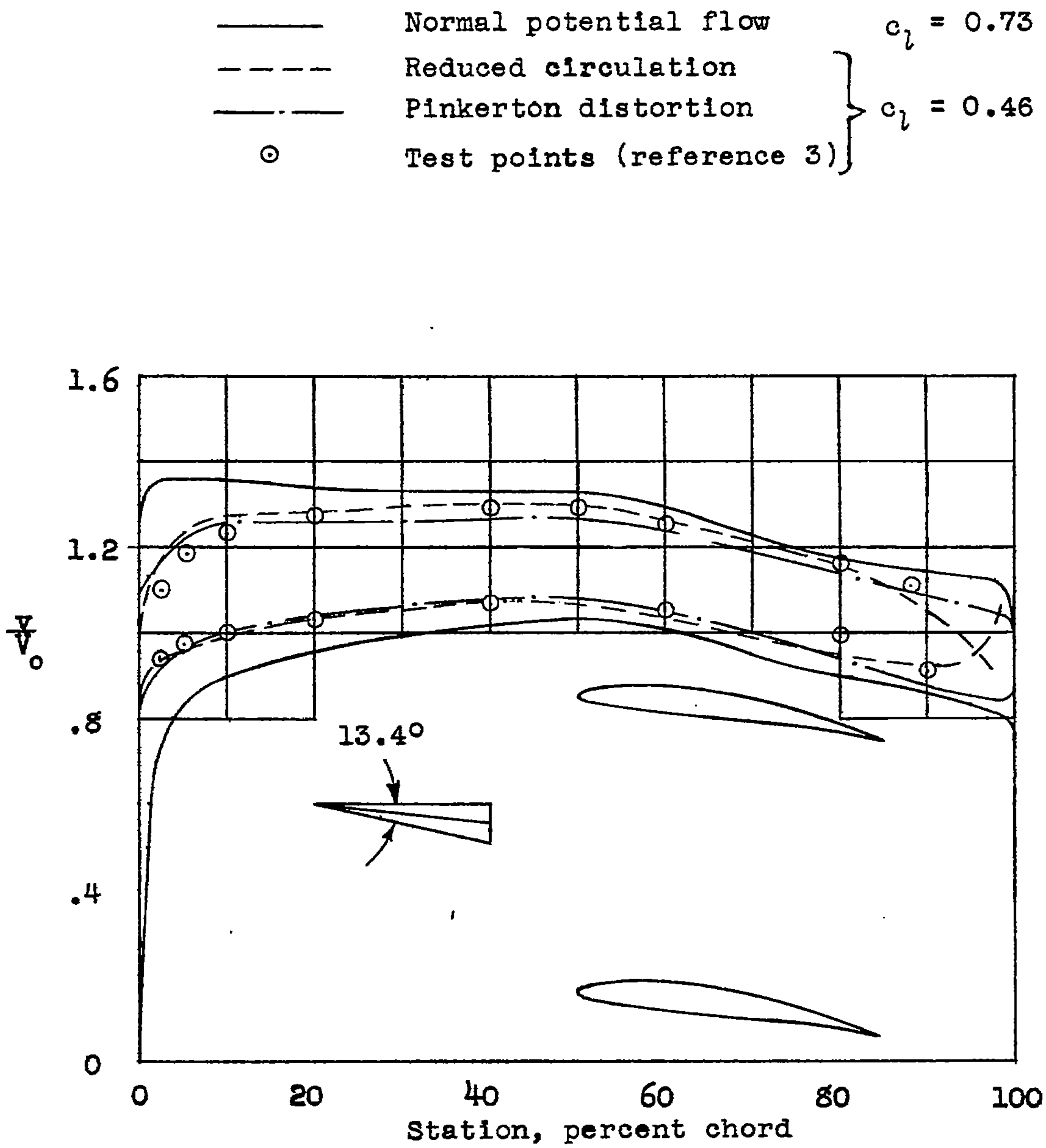

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Figure 1.- Comparison of experimental and theoretical velocity distribution on airfoil in entrance-vane cascade. NACA 65-810 airfoil; $\alpha_o = 1.8^{\circ}$; $\lambda_o = -6.8^{\circ}$; $\beta = 0^{\circ}$; $\sigma = 1.0$.

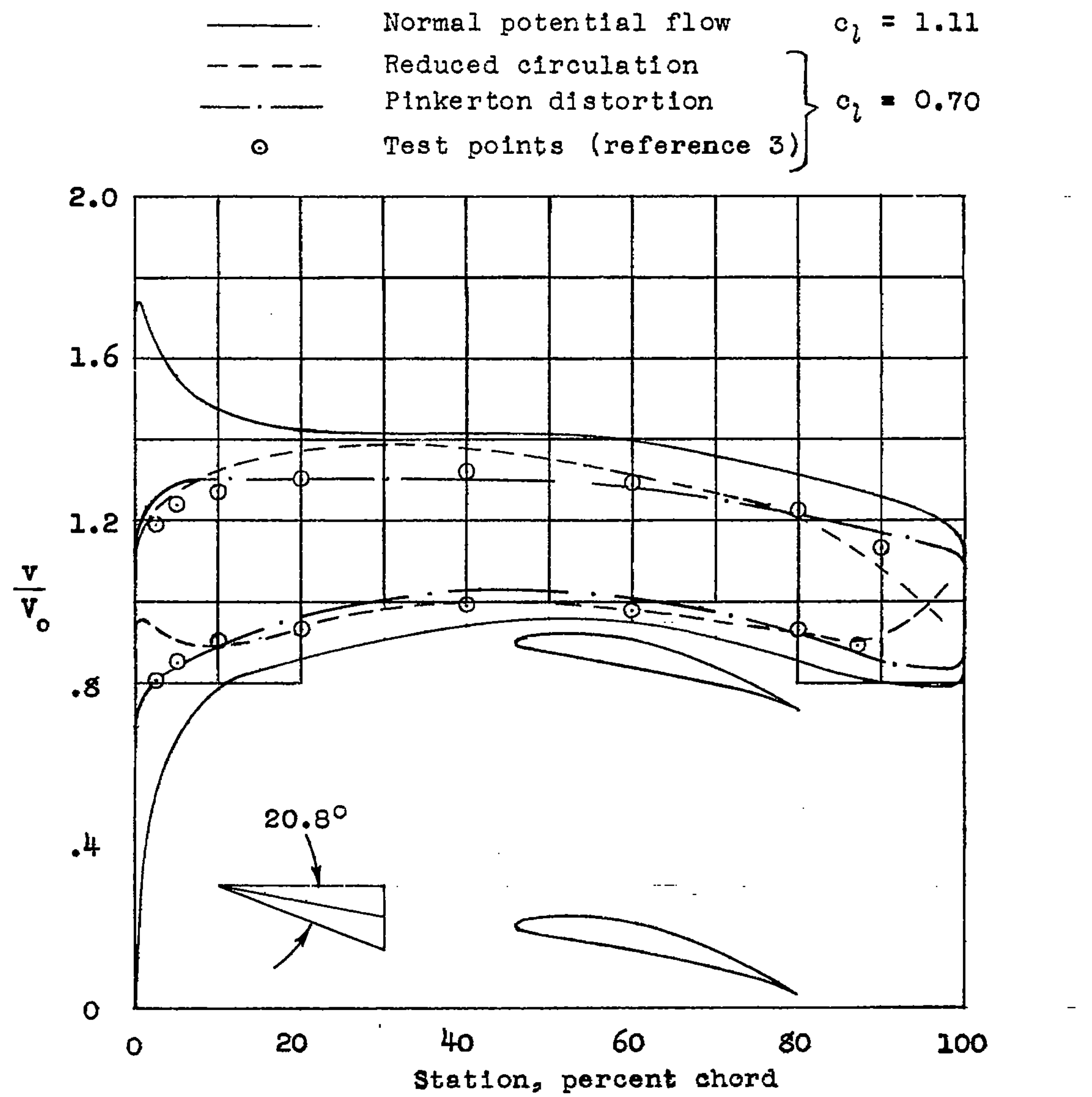

Figure 2.- Comparison of experimental and theoretical velocity distribution on airfoil in entrance-vane cascade. NACA 65-(12)10 airfoil; $\alpha_o = 3.4^\circ$; $\lambda_o = -10.8^\circ$; $\beta = 0^\circ$; $\sigma = 1.0$.

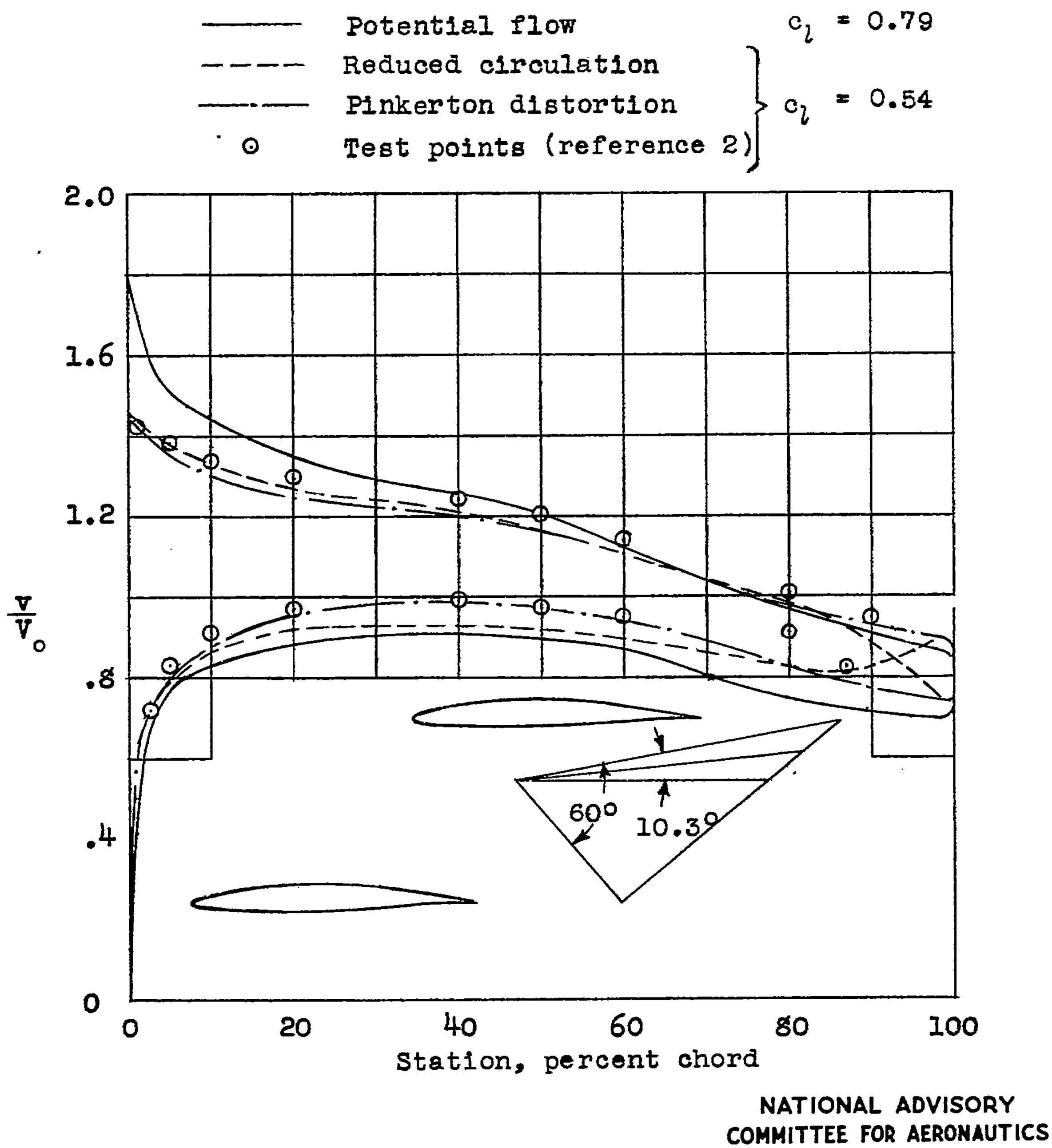

Figure 3.- Comparison of experimental and theoretical velocity distribution on airfoil in blower-blade cascade. NACA 65-410 airfoil; $\alpha_o = 6.5^o$; $\lambda_o = 55.5^o$; $\beta = 60^o$; $\sigma = 1.0$.

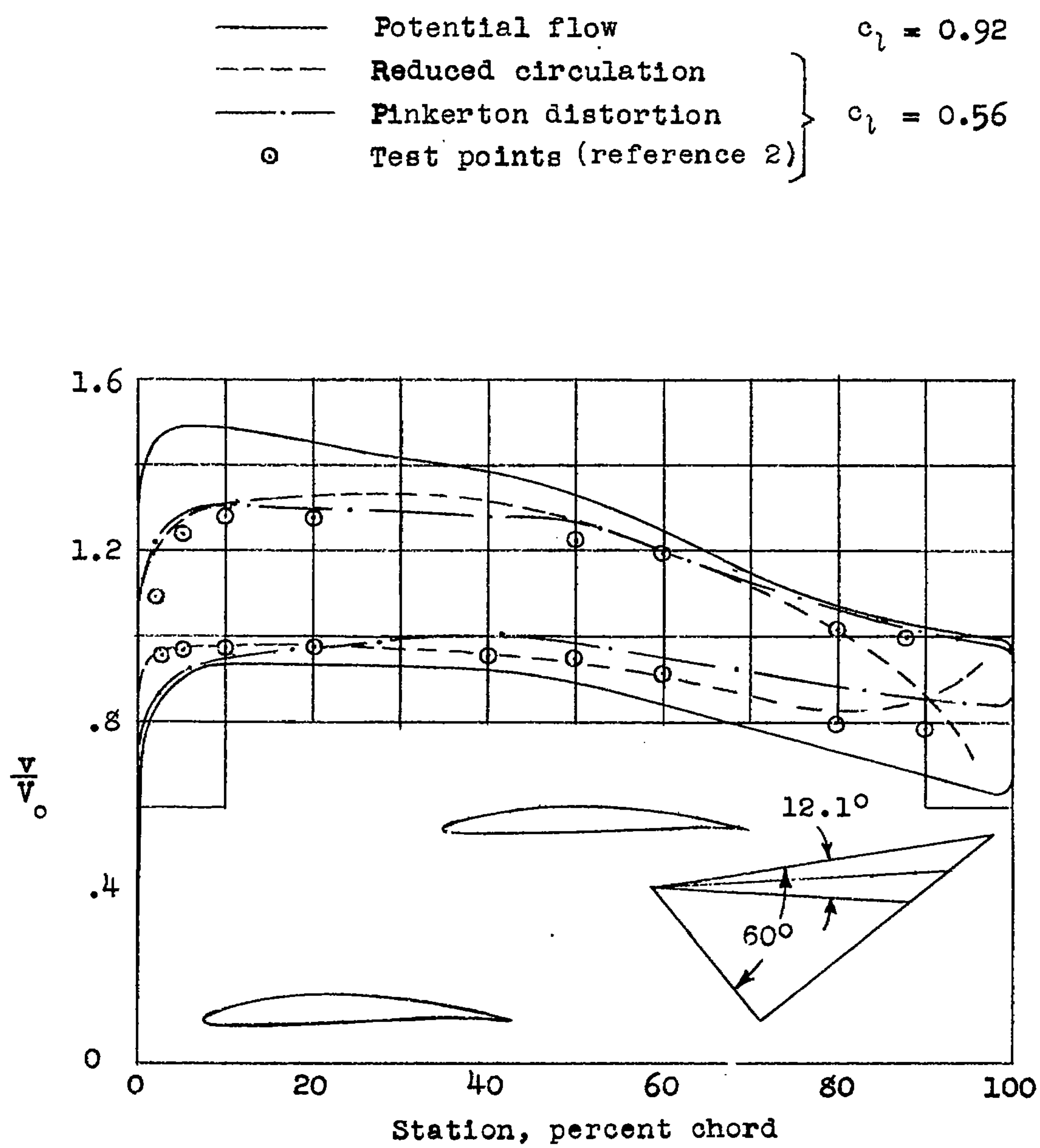

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Figure 4.- Comparison of experimental and theoretical velocity distribution on airfoil in blower-blade cascade. NACA 65-810 airfoil; $\alpha_o = 4.4^{\circ}$; $\lambda_o = 54.8^{\circ}$; $\beta = 60^{\circ}$; $\sigma = 1.0$.

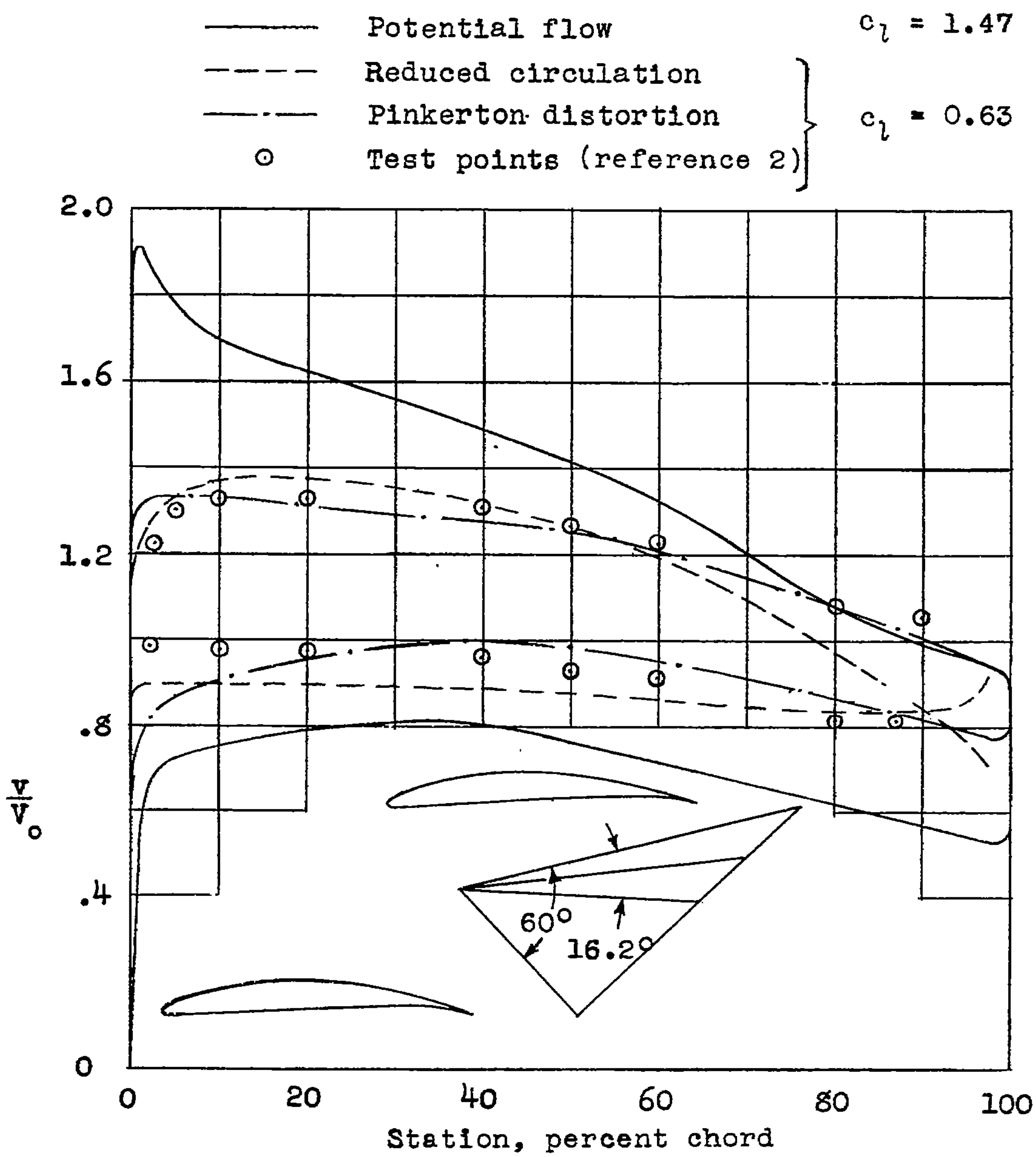

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Figure 5.- Comparison of experimental and theoretical velocity distribution on airfoil in blower-blade cascade. NACA 65-(12)10 airfoil; $\alpha_o = 7.5^{\circ}$; $\lambda_o = 53.4^{\circ}$; $\beta = 60^{\circ}$; $\sigma = 1.0$.

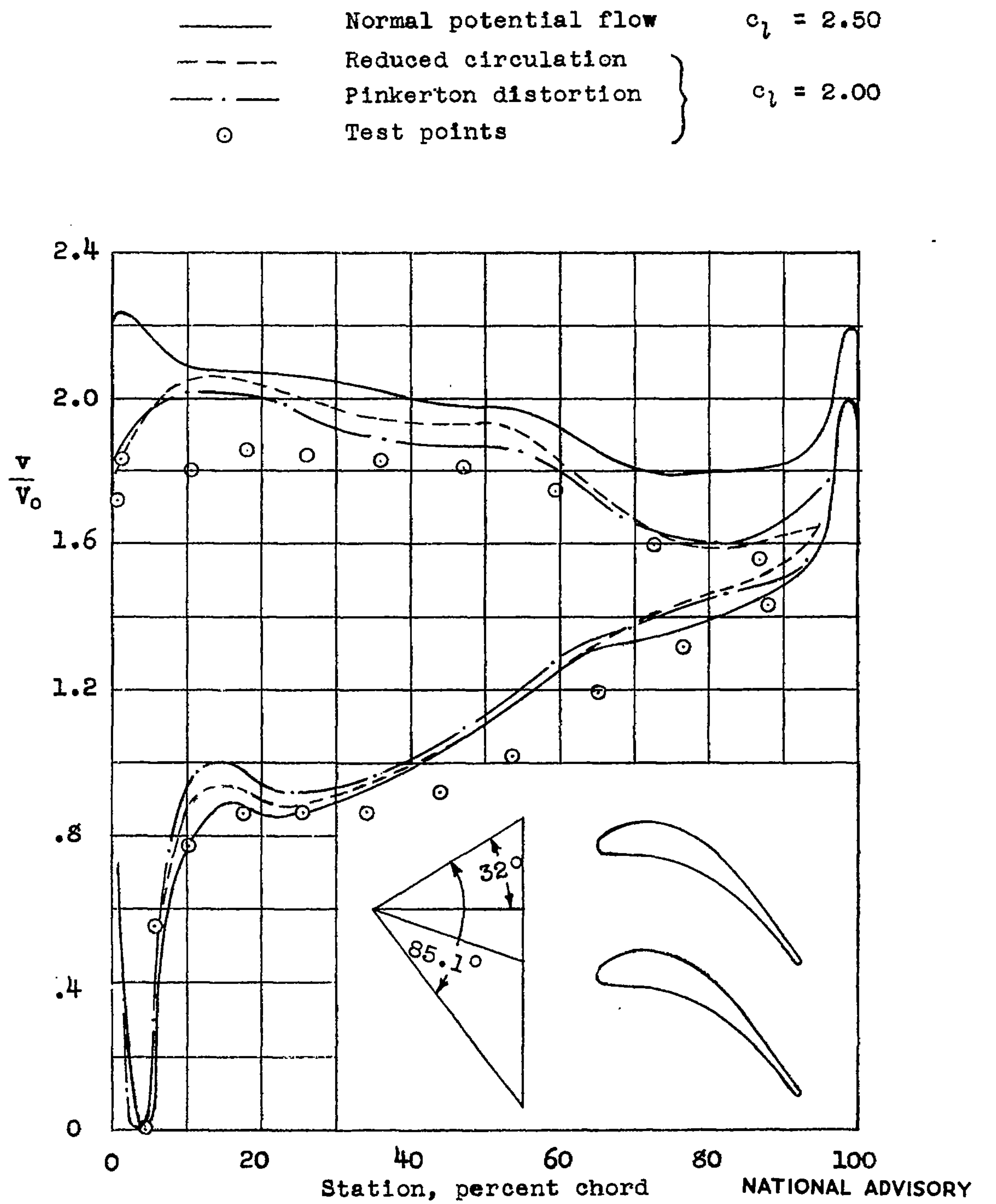

Figure 6.- Comparison of experimental and theoretical velocity distribution on turbine blade in cascade. $\alpha_o = 10.2^\circ$; $\lambda_o = -19.5^\circ$; $\beta = 32^\circ$; $\sigma = 1.8$; from unpublished data.

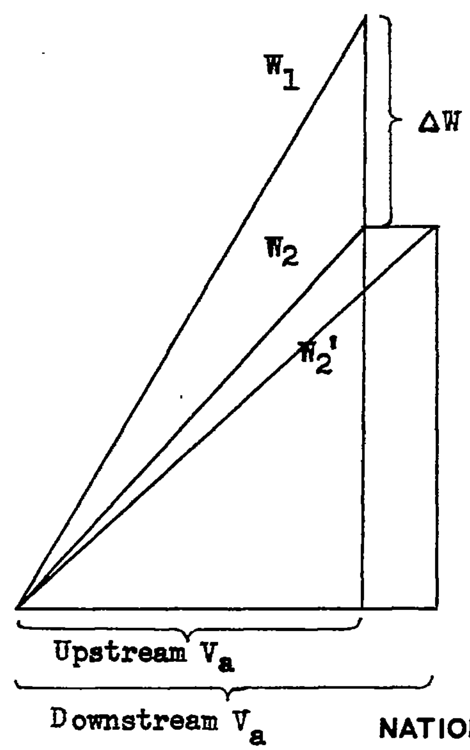

Figure 7.- Velocity diagram showing how, for given change in longitudinal velocity ΔW, the turning angle is affected by a downstream increase in axial velocity V_a. W_2 corresponds to constant V_a; W_2' results with increased downstream V_a.